ITM 207: Boolean Basics

BOOLEAN EXPRESSIONS, TRUTH TABLES, GATES

What is Boolean?

Boolean Variable - Variables that can only take on the values of either True or False; True is represented by 1 and False is represented by 0.

Boolean Expression - A mathematical notation that uses boolean logic and variables to express two-valued logic. Boolean Expressions use the following notations:

Boolean Operators:

- AND - Connects two boolean variables. The result will only be True if both variables are True. Notated with multiplication.
    - Example: A·B means A AND B;
- OR - Connects two boolean variables. The result will be True as long as one variable is True. Notated with addition.
    - Example: A + B means A OR B;
- NOT - Returns the complementary value of the attached variable. Notated with a prime symbol.
    - Example: A′ means NOT A

Tip: If you have previously taken SSH105, you can relate these boolean operators back to the logical connectives of conjunction, disjunction, and negation.

Truth Table - A table displaying all possible input values and the resulting outputs in regards to a boolean expression or circuit.

Expression: X = ABC + B′C′

Input columns →

	A	B	C	X
0	0	0	0	1
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

Notice how each row of input forms a binary number, and how the number increases down the table to display all possible combinations of the inputs!

→ Output column: Shows the output as the result of the input in each row

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Logic Gates

Gate - Devices that use electrical signals to perform basic operations. They can be combined to form circuits that can perform more complex tasks. The following are the gates covered in this course.

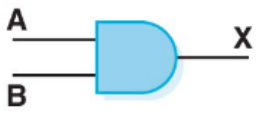
AND Gate:

- Outputs 1 if all input signals are 1, otherwise output 0
- Same as the AND operator
- SSH interpretation: The statement “Both A and B are true” is only true if both variables are true

Boolean Expression

$X = A \bullet B$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

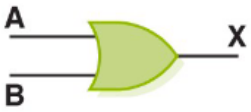
OR Gate:

- Outputs 1 as long as one input signal is 0, otherwise output 0
- Same as the OR operator
- SSH interpretation: The statement “Either A or B is true” is true as long as one of the variables is true

Boolean Expression

$X = A + B$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Note: 1 + 1 in boolean logic will yield 1. This is due to boolean data having only the possible values of 0 or 1. The output of an expression will always be either True or False, not more True or more False.

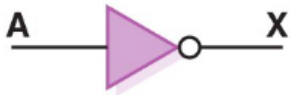
NOT Gate:

- Outputs the complement signal of the input
- Same as the NOT operator
- SSH interpretation: The statement “A is not true” is only true if A is false

Boolean Expression

$$X = A'$$

Logic Diagram Symbol



Truth Table

A	X
0	1
1	0

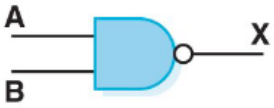
NAND Gate:

- Outputs 0 if all input signals are 1, otherwise output 1
- Means **N**OT **AND**, and is therefore the complement of the AND gate
- SSH interpretation: The statement “It is false that A and B are both true” is true when as least one of the variables is false

Boolean Expression

$$X = (A \cdot B)'$$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate:

- Output 0 as long as one input is 1, otherwise output 1
- Means **N**OT **OR**, and is therefore the complement of the OR gate
- SSH interpretation: The statement “Neither A or B is true” is only true when both variables are false

Boolean Expression

$$X = (A + B)'$$

Logic Diagram Symbol

Truth Table

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

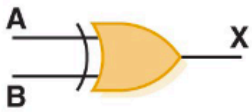
XOR Gate:

- Output 1 only if one input is 1, otherwise output 0
- Means e**X**clusive **OR**, the expression can also be shown as AB′ + A′B
- SSH interpretation: The statement “Only A is true or only B is true” is true when only one of the variables is true

Boolean Expression

$X = A \oplus B$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Tip: When memorizing the gates, focus on the expressions and the diagram symbols. While the truth tables are extremely helpful for understanding the relationship between the inputs and outputs, they can be quickly reconstructed by substituting the variables in the expressions with the possible combinations of 0s and 1s.